

## Extremal Properties on Bipartite Networks

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*Networks* describe systems in nature where the system components interact through some process. These components are described by *nodes* (vertices, people, computers) in a network and the interactions are described by the *links* (edges, connections, cables) of the network.

Many real-world networks, as social networks, display a bipartite structure. *Bipartite networks* consist of two groups or types of nodes, usually called *top* and *bottom*, with links only between vertices of different types (top with bottom nodes). For example the bottom set of nodes is people and the top is buildings in a city. One can construct the bipartite graph by connecting people with buildings, such that if a person visits a building, there is a link connecting them.

On the other hand, from the bipartite network one can construct the *unipartite projection network* into one of the sets of nodes. Projecting into bottom (top), two bottom (top) nodes are connected if both are connected to the same top (bottom) node in the bipartite graph. For example, two persons are connected (in the unipartite projection) if they visit the same building. This is clearly another social network that can be extracted from the bipartite structure.

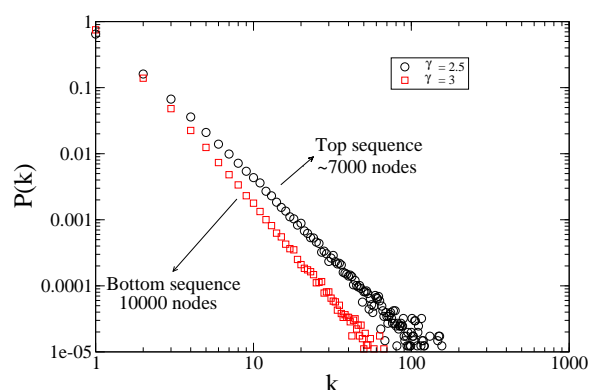
By studying the projected network structure we could find extremal properties. This is, given two degree sequences (set of positive integer numbers representing the degree of each node in the set), how to construct a bipartite network that maximizes a given property of the projected network?

In this project we found the maximal clustering coefficient and assortativity on projected networks by choosing the right wiring model in the bipartite graph. In particular we used two power-law degree sequences (see figure) and constructed the bipartite network through three wiring models:

**Configuration Model:** Create edges between two randomly chosen vertices in top and bottom.

**Havel-Hakimi Model:** Create edges between the highest degree nodes in top and bottom.

**Havel-Hakimi Reverse Model:** Create edges between the highest degree node in top and the lowest degree node in bottom.



*Degree distribution of Top and Bottom degree sequences used to construct the bipartite network using various wiring models.*

Model	Average Clustering	Assortativity
Configuration Model	0.63238	0.23582
Configuration Model No-Parallel Edges	0.63507	0.22775
Havel-Hakimi Model	0.35861	0.01617
Havel-Hakimi Reverse Model	0.84873	0.99363

*Results obtained for clustering coefficient and assortativity of the projected network using different wiring models.*

In the table we show that the model that maximizes the clustering coefficient and assortativity of the projected network is the Havel-Hakimi Reverse Model. This happens because this model optimizes (maximizes) the number of edges and tends to connect the maximum number of nodes in the projected network giving the highest number of triangles and, as a consequence, maximizing the clustering coefficient. Moreover, because this model tends to generate a complete graph in the projected network, the assortativity becomes large.

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